Misconception In Circular Motion

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Abstract - Centripetal and centrifugal are the forces which are equal in magnitude and opposite in direction, so how can a body revolve **in** circular orbit where there can be chances of cancellation of centripetal and centrifugal forces? This problem can be solved if we consider the direction of resultant of these two forces provided that system is non- relativistic system, magnitude of torque acting on system is 0 and there is no dissipation of energy.

Index Terms –Centripetal force, Centrifugal force, Limits, Torque ,Vectors, Infinite series

1 Introduction

As we know that,when a body having constant angular velocity performing uniform circular motion revolves continuously, now t he question arises that how can a body revolve in circular orbit where there can be chances of cancellation of centripetal and centrifugal forces? If the centripetal and centrifugal forces keep the body in circular orbit, then how this phenomenon is achieved and if we just say if centripetal is action, then according to Newton's law of motion , centrifugal is reactive force , hence they don't cancel each other.Now the question arises that if they don't cancel each other then how the resultant of these forces keep the body in circular orbit? To find the solution of this problem we have to use the vector method and Limits.

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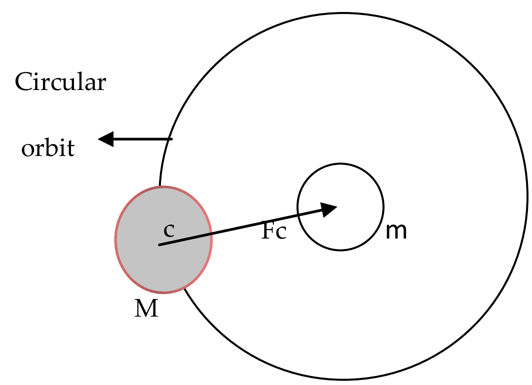
2 Theory



Consider a body in space having mass 'M' revolving around a body having mass 'm' in a circular path.

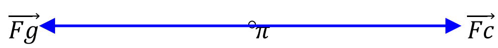
Since body having mass 'M' is in circular motion, it will have some centripetal force say 'Fc' which is directed towards the centre i.e. to towards body having mass 'm'. Body 'M' will also have centrifugal force say 'Fg' which is equal in magnitude and opposite in direction i.e. $\overrightarrow{Fc} = - \overrightarrow{Fg}$.

Both Fc and Fg act along the same line but opposite in direction i.e. they could be

1>opposite vectors or

2> anti parallel vectors.

Since Fc and Fg are along the same line then these vectors must be anti parallel as shown in fig. below



Now angle between Fc and Fg is π^c .

To find resultant of Fc and Fg we have to use law of parallelogram of vectors i.e.

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

Here P=Fc=Q=Fg, θ=π.

$$R = \sqrt{Fc^2 + Fc^2 + 2Fc^2\cos\pi}$$

$$R = \sqrt{2Fc^2 - 2Fc^2}$$

$$R = \sqrt{0} = 0.$$

Now regarding about direction of $\vec{R}$ which is given by,

$$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta}$$

Here θ=π^c, P=Q=Fc=Fg.

$$\tan\alpha = \frac{Fc \cdot \sin\theta}{Fc + Fc \cdot \cos\theta}$$

$$\therefore \tan\alpha = \frac{\sin\theta}{1 + \cos\theta}$$

taking limits on both sides as $\theta \to \pi^c$

$$\therefore \lim_{\theta\to\pi^c} \tan\alpha = \lim_{\theta\to\pi^c} \frac{\sin\theta}{1 + \cos\theta}$$

$$\therefore \tan\alpha = \lim_{\theta\to\pi} \left[\frac{2\sin\frac{\theta}{2} \cdot \cos\frac{\theta}{2}}{\frac{2\cos^2\frac{\theta}{2}}{1}} \right]$$

$$\therefore \tan\alpha = \lim_{\theta\to\pi} \tan\frac{\theta}{2}$$

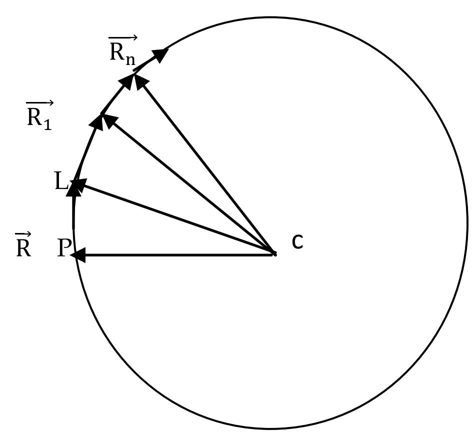
$$\tan\alpha = \tan\frac{\pi}{2}$$

$$\therefore \alpha = \tan^{-1} tan\,\frac{\pi}{2}$$

$$\therefore \alpha = \frac{\pi}{2}$$

Hence we can find the direction of $\vec{R}$ *which is* α.

Now circle is a locus of many (infinite) equidistant points as shown in fig. below,



Now in above above figure, c is centre of circular orbit and a body is revolving in that path. Here angle between seg PC and $\vec{R}$ is π. now if we move towards next point L then angle between OL and $\vec{R}_1$ is π.thus as we move towards next points we get a complete circular path. Here magnitude of $\overrightarrow{R_1}$,$\overrightarrow{R_2}$...... $\overrightarrow{\mathrm{R_n}}$ ... ∞ , is zero but their direction is perpendicular to line joining centre and the point on circular orbit .since body can't revolve in this directed path by resultant vectors because their magnitude is zero hence the body should have some velocity so that it can revolve in the circular path. Hence initially corresponding velocity should be given to particle tangentially so that it can revolve in circular path with constant angular velocity.

Now one may say that does the velocity has to have specific value depending upon the magnitude of radius vector? To find it's solution let us consider the series given below,

$$\frac{dR_1}{dt}+\frac{dR_2}{dt}+\frac{dR_3}{dt}+\cdots+\frac{dR_n}{dt}+\cdots\infty=\sum_{n=1}^{\infty}\frac{dR_n}{dt}$$

Where $\frac{dR_1}{dt}$ is the rate of change of change of magnitude of R_1 with respect to time t similarly $\frac{dR_2}{dt}$ is the rate of change of R_2 with respect to time t ,and so on..

Now consider the above series i.e. $\sum_{n=1}^{\infty}\frac{dR_n}{dt}$,if the limiting of this series is finite then it is convergent. To find it's limiting value we will use Cauchy's integral series test.

Here f(t)=$\frac{dR_n}{dt}$.

$$\therefore \int_1^{\infty} f(t)dt = \lim_{m\to\infty}\int_1^{m}\frac{dR_n}{dt}\,dt$$

$$=\lim_{m\to\infty}[R_n]_1^m$$

$$=\lim_{m\to\infty}[R_m - R_1]$$

$$= R_\infty - R_1$$

$$= 0$$

∴ (Here $\overrightarrow{R_1}$,$\overrightarrow{R_2}$...... $\overrightarrow{\mathrm{R_n}}$ ... $\infty = 0$)

Which is finite value , hence if and only if $\lim_{n\to\infty}\sum_{n=1}^{\infty}\frac{dR_n}{dt}=0$ and if above series is convergent then and then only we can apply the above theory of finding the direction of centripetal and centrifugal forces using vector method. This is because if $\lim_{n\to\infty}\sum_{n=1}^{\infty}\frac{dR_n}{dt}\neq 0$ then the motion of object revolving in orbit would not be uniform circular motion, and hence we can't apply this theory in that case.

Now revolving a body in circular orbit needs initial velocity and this can be achieved by calculating magnitude of radius vector and it's corresponding velocity by our conventional method.

Here the question arises that the body is also performing rotational motion hence torque must be there so does it affect the resultant of centripetal and centrifugal forces? The answer is, since the body has constant angular velocity i.e. $\frac{dw}{dt}=0$

$\therefore \tau_{net} = I\frac{dw}{dt}$ = 0. And this proof relies on the assumption that mass is constant; this is valid only in non-relativistic system in which no mass is being ejected. It also tells us that the law of conservation of moment is conserved.

The next point is, is there any dissipation of energy when the body is revolving with constant angular velocity?

As we know that E= $\tau\theta$, *where* τ is magnitude of torque and θ is the angle moved(in radians). Since torque is 0, 'E' will also equal to 0. Hence there is no dissipation of energy.

3 Conclusion

The conclusion to this topic is if a body is revolving with a constant angular velocity then magnitude of resultant of these forces is 0, but the direction of resultant of these forces keep the body in circular orbit provided that system is non-relativistic system, magnitude of torque acting on system is 0 and there is no dissipation of energy and $\lim_{n\to\infty}\sum_{n=1}^{\infty}\frac{dR_n}{dt}$ should equal to 0.

4 Acknowledgments

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5 References

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